IN THE SPECIFICATION:

(1) Kindly replace the paragraph beginning on page 10, line 4 with the following paragraph:

--The geometry of the nets C1, C2, C3 is described hierarchically and captured to a level of detail needed to determine an accurate solution. For example, in FIGURE 1, the net C1 and the net C2 are captured with relatively detailed geometric descriptions. However, the net C3 may be captured with relatively coarse geometric descriptions.--

(2) Kindly replace the paragraph beginning on page 7, line 8 with the following paragraphs:

--FIGURE 2A illustrates a block diagram of a system for determining a capacitance of an integrated circuit constructed according to the principles of the present invention;

FIGURE 2B illustrates a flow chart of a method of determining a charge distribution for a net constructed according to the principles of the present invention;--

(3) Kindly replace the paragraph beginning on page 11, line 5 with the following paragraph:

--Turning now to FIGURE 2A, illustrated is a block diagram of a system for determining a capacitance of an IC, generally designated 200, constructed according to the principles of the present invention. The system 200 comprises an integral equation formulator 210, a charge variation function generator 220, and a conductive geometry generator 230. The integral equation formulator 210 determines the total capacitance of all the nets C1, C2, C3 within the IC.--

(4) Kindly replace the paragraphs beginning on page 13, line 20 and ending on page 24, line 10 with the following paragraphs:

--In one embodiment of the present invention, the characteristic geometry function for the set of net surfaces, χ_R, is a predefined function. In an alternate embodiment of the present invention, χ_R is calculated from information related to the capacitive structure of the IC. χ_R is scalar-valued function that has a n-dimensional position as the input. To illustrate the form of χ_R, suppose the geometry to be described is the unit square in the *xy*-plane. The form of χ_R for this geometry then would be:

$$\chi_R = s(x)s(y)\delta(z), \qquad (1)$$

where s(x)=1 for $0 \le x \le 1$ and s(x) is zero elsewhere, and d(z) is the standard Dirac delta function. Those skilled in the art are familiar with the properties and use of Dirac delta functions.

The function f, which represents the charge distribution without regard to geometry, is evaluated over R, the set of surfaces of the nets C1, C2, C3. The surface integrals over R are reformulated as volume integrals via the relation:

$$\int_R f = \int_V f\chi_R \qquad (2)$$

where V is the volume that the net comprises.

To solve the iterative linear solution, the integral equation formulator 210 uses an initial guess g, for the charge distribution, and a starting subdivision of the geometry of the net. The integral equation formulator 210 also creates the projection matrix P_R from χ_R.

$$P_R = \sum_k \langle \chi_R, l_k \rangle P_k \qquad (3)$$

The bracketed expression above is the standard inner product for functions:

$$\langle f, g \rangle = \int_V fg \qquad (4)$$

Also in this equation, P_k is a primitive projection matrix where the (i,j) entry of $P_k = \int_V l_i l_j l_k$. Finally the l_is represent the choice of orthogonal polynomials. In one embodiment of the present invention, Legendre polynomials form the system of orthogonal polynomials. One skilled in the pertinent art is familiar with and the use of Legendre polynomials. Also, P^b_R will represent that part of P_R that contains information of the geometry of subdivision b.

Using the initial guess g for the charge distribution and the starting subdivision of the geometry of the net, the integral equation formulator 210 determines charge distributions for a given potential on the nets C1, C2, C3. From the charge distributions, the integral equation formulator 210 determines the capacitance of the nets C1, C2, C3.

In the determination of the charge distributions, the iterative linear solution uses the following fundamental equation that relates the charge and the potential:

$$\psi(r) = \int_R G(r,r')\rho(r')dr' \qquad (5)$$

where G (r, r') is the Green's function for the geometry of the IC and ρ(r') is the charge density. Those skilled in the art are familiar with the properties and use of Green's functions.

In one embodiment of the present invention, the fundamental equation relating the charge and the potential is expressed as:

$$\psi = MP_R f \qquad (6)$$

where the ψ is a potential distribution and f represents the charge distribution without regard to the geometry. From this equation, the integral equation formulator 210 can determine the charge distribution function f.

In one embodiment of the present invention, the integral equation formulator 210 uses a Krylov method based on a variation of a Generalized Minimal Residual Method (GMRES) for the iterative linear solution. Background information concerning GMRES is discussed in GMRES: A Generalized Minimal Residual Algorithm for Solving Nonsymmetric Linear Systems, by Y. Saad and M. H. Shultz, SIAM Journal on Scientific and Statistical Computing, 7(3):856-69, 1986 (incorporated herein by reference).

GMRES is used to solve for the charge distribution function f in the following equation:

$$\left\| MP_R f - \psi_0 \right\|_R = 0 \qquad (7)$$

or, equivalently:

$$P_R M P_R f = P_R \psi_0, \qquad (8)$$

where M is an operator that converts a charge density to a potential distribution and ψ_0 is a potential which is 1 over an individual net and 0 elsewhere. Those skilled in the pertinent art are familiar with the application of GMRES. Nevertheless, below is a description of the process both to give a

complete picture of the iterative linear solution process and to show the advantages of decoupling the charge variation from the geometry.

The integral equation formulator 210 starts iterative linear solution with an initial guess g for the charge distribution and an initial geometry. The integral equation formulator 210 also determines χ^b_R, which represents the geometry of the box b in the subdivision, and P^b_R, which is the part of the projection matrix P_R which covers the box b (see FIGURE 3A for more information concerning geometry subdivisions and boxes). From this information, the integral equation formulator 210 computes a potential ψ based upon the initial charge distribution guess g and the current subdivision.

The integral equation formulator 210 also determines the desired potential ψ_0:

$$P_R\psi_0 = P_S o \qquad (9)$$

where o is the constant function 1 over an individual net and P_S is defined as P_R over an individual net S. The integral equation formulator 210 then determines the difference between the desired potential ψ_0 and the potential ψ. This difference, ψ_0-ψ, becomes first charge variation function f_1. Note that f_1 is not a representation of the charge distribution described by f. f_1 is a function used to modify the charge distribution function f to reach a desired resolution.

The integral equation formulator 210 determines the ratio γ/β, where $\gamma = \|f_1\|R$ and $\beta = \|\psi_0\|R$. If the ratio is sufficiently small, the iterative linear solution is complete.

If the ratio is unacceptably large, the integral equation formulator 210 normalizes f_1 and proceeds with the linear iterative solution. The integral equation formulator 210 then invokes the charge variation function generator 220 to create a charge variation function which refines the description of the charge distribution. At the beginning of the iterative process the charge variation

function generator 220 creates a second charge variation function f_2. In general, the formula for the creation of the $j+1^{th}$ charge variation function from the j^{th} charge variation function is:

$$f_{j+1} = MP_R^b f_j \quad . \qquad (10)$$

Where M is an operator that converts a charge density to a potential distribution. The (i,j) entry of the matrix representation of M is:

$$M^{bc} = \int_b \int_c l_i^b l_j^c G(r,r')dr'dr \qquad (11)$$

Where l^b_i is the i^{th} moment in box b. Note that the operation of M involves all boxes, not just box b. G(r,r') is the Green's function for the set of net surfaces R. Those skilled in the art are familiar with the properties and use of Green's functions.

The charge variation function generator performs the operation M using a variant of the Fast Multipole Method (FMM) algorithm called the Fast Distributed Method (FDM). The FDM differs from the FMM in several areas. First, the FDM omits the FMM's initial step of computing multipole representations from point charges. FDM omits this step because the input is already in terms of a charge distribution. Second, the FDM omits FMM's direct point-to-point interactions. Instead, the FDM uses a source charge distribution in a cube. The potential distributions are calculated in all neighboring cubes, including the source cube itself.

Third, the FDM omits FMM's final step of interpolating point potentials from the local expansions. The FDM omits the last step, because the desired output is a potential distribution. Finally, FDM uses Legendre polynomial expansions for both charge distributions and local

expansions. The FMM uses multipole expansions for representing charge distributions and local expansions for representing potential distributions.

The FMM algorithm, Legendre polynomial expansions and their use are well known to those skilled in the pertinent art. Background information concerning numerical analysis and capacitance calculations is discussed in Introduction to Numerical Analysis, by J. Soer and R. Bulirsh, Springer-Verlag 1979 and in Preconditioned, Adaptive, Multipole-Accelerated Iterative Methods for Three-Dimensional First-Kind Integral Equations of Potential Theory, by K. Nabors, *et al.*, SIAM Journal on Scientific and Statistical Computing, 15(3):713-735, May 1994 (both incorporated herein by reference).

The use of the FDM results in a significant savings in time over both the analytical calculation using the explicit form of M and the FMM. The savings are due to the omission of several FMM steps and the omission of FMM's direct point-to-point interactions.

Once the charge variation function generator 220 has created the charge variation function f_{j+1}, the integral equation formulator 210 uses this new function to create the (i, j) entry of a Hessenberg matrix:

$$H_{ij} = \left\langle f_i, f_{j+1} \right\rangle_R \qquad (12)$$

for all $i \leq j$.

The integral equation formulator 210 uses the Hessenberg matrix to solve for the new coefficients of the charge variation functions to be used to further refine the charge distribution function f.

After the entries in the Hessenberg matrix are computed, the integral equation formulator 210 then orthogonalizes f_{j+1} from the other f_is. The charge variation function generator 220 then creates the (j+1,j) entry of the Hessenberg matrix by the expression:

$$H_{j+1,j} = \left\| f_{j+1} \right\|_R . \qquad (13)$$

This expression is the standard norm of functions defined by:

$$\left\| f \right\|_R = \sqrt{\langle f, f \rangle} = \sqrt{\int_R f^2} \qquad (14)$$

After the creation of this last entry of the Hessenberg matrix, the integral equation formulator 210 then normalizes f_{j+1}. The orthogonalization and the normalization of f_{j+1} create orthonormal charge variation functions. As a result, the integral equation formulator 210 is able to obtain the correct charge distribution function f very quickly.

The integral equation formulator 210 then solves for **x**, the coefficient vector for the charge variations, using least-squares on the equation:

$$Hx = \gamma e_1 \qquad (15)$$

where H is the Hessenberg matrix generated by the integral equation formulator 210 as explained above. $\mathbf{e}_1$ is the 1st unit vector with the form (1,0,0,0,0, . . .)(with j number of zeros). The vector **x** is the coefficient vector for the charge distributions and γ is the norm of the first charge variation function. γ is defined as:

$$\gamma = \left\| f_1 \right\|_R . \qquad (16)$$

Since the charge variation and the geometry are decoupled, the representation of the f_i will not become very large as compared to the previous methods. Using the residual of the least squares, the integral equation formulator 210 computes the factor r as the 2-norm of the least squares residual.

The integral equation formulator 210 then generates a new guess for the charge distribution function f using the equation:

$$f = g + \sum_{i=1}^{j} x_i f_i \qquad (17)$$

The integral equation formulator 210 then determines the ratio r/β. Where r is the 2-norm of the least squares residual r and β is the norm of the desired potential (defined earlier as $\beta = \|\psi_0\|R$). If the ratio is within acceptable limits, the charge distribution has converged and the integral equation formulator 210 terminates the iterative linear solution.

If the ratio is not within acceptable limits, the integral equation formulator 210 uses the new guess for the charge distribution function f, calculated above, as the starting point for the next iteration of the iterative linear solution. However, before the integral equation formulator 210 performs the next iteration, the integral equation formulator 210 must determine if the current subdivision requires a more refined approximation.

The integral equation formulator 210 determines if the current subdivision b requires refinement by calculating a charge-geometry error e. In one embodiment of the present invention, the integral equation formulator calculates the charge-geometry error e^b associated with box b in the subdivision using the following equation:

$$e^b = \sum_{i=k}^{N-1} f_i^b l_i \qquad (18)$$

Where l_i is the i^{th} moment basis function. f represents the distribution of the charge without any reference to the geometry of the IC and f_i^b is the component of f along the i^{th} moment for the charge function contained in the subdivision b. The summation of l_i between k and N-1 represent the high order polynomials. Finally, N is the number of moments used in the expansion of f.

The integral equation formulator 210 then determines if the charge-geometry error is within acceptable limits using a charge-geometry error criterion. The charge-geometry error criterion is defined as:

$$\|e^b\|_R < \frac{\varepsilon}{\|f\|_S} \qquad (19)$$

Where S is the minimal subdivision containing the selected net. ε is a small constant that determines the allowable tolerance of the criterion. Also, $\|e^b\|_R$ is the magnitude of the charge-geometry error in subdivision b. Note that the error threshold is relative to the charge distribution f only on the subdivision of the selected net.--

(5) Kindly insert the following paragraphs after the paragraph that ends on 24, line 10:

--Turning now to FIGURE 2B with continued reference to FIGURE 2A, illustrated is a flow diagram of an embodiment of a method of determining a charge distribution for a net, generally

designated 250, constructed according to the principles of the present invention. The method starts in a step 255 with an intent to determine a charge distribution.

An initial charge distribution and geometry are provided in a step 260. The initial charge distribution and the initial geometry may be guesses and are used to start an iterative linear solution. The initial guess for the charge distribution may be designated g and the initial guess for the geometry may be a subdivision of the geometry of the net.

After an initial charge distribution and geometry are provided, a first charge variation function is then determined in a step 265. The first charge variation function may be the difference between ψ_0 and ψ. In one embodiment, the first charge variation function may be determined by solving for ψ_0 and ψ using Equations 6 and 9.

After determining the first charge variation function, a determination is made if the charge distribution function is within an acceptable limit in a first decisional step 270. In one embodiment, the acceptance of the charge distribution function may be within an acceptable limit if the ratio γ/β is sufficiently small. If the charge distribution function is within an acceptable limit, the method 250 ends in a step 295.

If the charge distribution function is not within an acceptable limit, the charge variation function is refined through an iterative linear method in a step 280. In one embodiment, f_1 may be normalized before proceeding with the linear iterative method. The method 250 creates a charge variation function which refines the description of the charge distribution employing Equations 10-17.

After refining the charge variation function, a determination is made if the charge distribution function is within an acceptable limit in a step 282. In one embodiment, the charge distribution

function is within an acceptable limit based on the ration r/β. If the ratio is within acceptable limits, the charge distribution has converged and the method 250 continues to step 295 and ends.

If the charge distribution function is not within acceptable limits, a determination is made if the geometry needs refinement in a third decisional step 287. In one embodiment, the method 250 may employ Equations 18 and 19 to determine if the geometry needs refinement. If the geometry does not need refinement, the method continues to step 280. If the geometry does need refinement, the geometry is subdivided into subdivisions in a step 290. After subdividing the geometry, the method 250 continues to the step 280.

While the methods disclosed herein have been described and shown with reference to particular steps performed in a particular order, it will be understood that these steps may be combined, subdivided or reordered to form an equivalent method without departing from the teachings of the present invention. Accordingly, unless specifically indicated herein, the order and/or the grouping of the steps are not limitations of the present invention.--